

Name _____

MTH 2001 - Calculus 3

PRACTICE ADVANCED STANDING EXAM

No outside resources are permitted including: notes, textbooks, cell phones or any other electronics. Show all work. Solutions without explanations will receive no points. Simplify your answers. Circle your final answers.

1. Calculate the given quantities using the following vectors:

$$\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

a. $2\mathbf{u} + 3\mathbf{v}$

b. $|\mathbf{u}|$ and $|\mathbf{v}|$

c. A unit vector going in the same direction as

d. $\mathbf{u} \cdot \mathbf{v}$

e. $\mathbf{u} \times \mathbf{v}$

f.

3.

6. Given the function $(x, y, z) = x^2 + y^3 + 2z^2$, where $x = \sin t$ and $y = t^2$ and $z = 3t + 2$

7. Find and classify all the Critical Points of the function $f(x, y) = 3x^2 - 2xy + 3y^2 - 6x - 6y + 12$ -

8. Find the Absolute Max and Absolute Min of the function $f(x, y) = x^2 - 2x + 2y$ on the triangular region in the xy -plane bounded by the points $(0,0)$ and $(2,0)$ and $(2,4)$

10. Evaluate the following integral by reversing the order of integration: $\int_0^1 \int_{-\frac{2}{3}}^1$

11. Find the mass of a thin metal plate that occupies a region D that is bounded by the parabola $y = 1 - x^2$ and the coordinate axes in the first quadrant if the density of the plate varies according to the density function $\rho(x, y) =$

12. Find the volume that lies inside the cylinder $x^2 + y^2 = 4$ and above the xy -plane and beneath the paraboloid $z = x^2 + y^2 + 1$ by using cylindrical coordinates.

13. Show that the volume of the upper half of a sphere of radius $\frac{2}{3}$ is $\frac{2}{3}$ by using spherical coordinates.

14. Evaluate the following integral: $\int_0^1 \frac{4x^2+1}{4x^2} dx$
by making the transformations: $u = 2x$ and $du = 2 dx$

15. Evaluate the following integral: $\int (x - 2) dx$ over the triangular region that has vertices at the points (0,0) and (1,2) and (2,1) by making the transformations: $u = 2x + y$ and $v = x + 2y$
(Note: You must show the appropriate work for a change-of-variable problem. You will not receive any credit if you attempt to leave the integral in its original x - y -form.)

16. Evaluate the following line integrals by parameterizing the curves:

8 where is the arc of the parabola $= t^2$ from $(0,0)$ to $(2,4)$

$3 + 2$ where is the arc of the parabola $= 1 - t^2$ from $(0, -1)$ to $(0,1)$

· where $= t + (t + 1)$ and is given by $(t) = t^2 + t^3 -$ for $0 \leq t \leq 1$

17. Use the Fundamental Theorem of Line Integrals to evaluate the following integral:

$$\int_C (2x + 2y + z) + (2x + y + z) + (2x + y + \cos(z))$$

where C is the line segment parameterized by the function $\mathbf{r}(t) = 4 + (2 - 2t) + 3t$ for $0 \leq t \leq 1$.

18. Use Green's Theorem to evaluate the integral $\int_C (2x^2 - y^3) dx + (2x^2 - y^3) dy$

19. Use Stokes' Theorem to evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ and C is the triangle with vertices at $(1,0,0)$ and $(0,1,0)$ and $(0,0,1)$ with counterclockwise orientation when viewed from above.

