

Name _____

MTH 1001 - Calculus 1

PRACTICE ADVANCED STANDING EXAM

1. (a) Write the general *definition of the derivative* for a function $f(x)$
[14pts]

(b) Find $f'(x)$ by using the *definition of the derivative* with the following function:

$$f(x) = \frac{1}{x}$$

2. Find the derivative: $f(x) = x^3 \tan 2x - 1$
[12pts]

3. Find the derivative: $f(x) = e^{x^3} \ln \sec x - \csc \ln x$
[9pts]

7. A toy car moves along a straight track during time $0 \leq t \leq 4$. Its position at any time from a fixed point along the track is given by $s(t) = t^3 - 3t^2$

[10pts]

Answer the following about the motion of the car.

(Note: The time t is measured in minutes and distance s in inches.)

(a) What is the position, velocity, and acceleration of the car at the time $t = 3$ minutes?

(b) At what time does the car come to a stop?

8. A 5 ft ladder is leaning against a wall and starts to slide. How fast is the bottom edge of the ladder moving along the floor when the top corner of the ladder is 3 ft up the wall and sliding down the wall at a rate of 8 ft/sec?

[12pts]

9. Use L'Hôpital's Rule to evaluate the following limit:

[8pts]

$$\lim_{x \rightarrow 0} \frac{x^3 - 5\sin x}{x \cos x}$$

10. Graph the following Rational Function:

$$f(x) = \frac{36x - 1}{x^2}$$

11. A box with a closed top is going to be manufactured so that its base is a square and its volume will be 100. If the material to make the top and bottom of the box cost \$50 per square cm and the material for the sides costs \$4 per square cm, find the dimensions that will minimize the cost of the box.

12. Find the exact area under the curve $f(x) = 2x - 1$ over the interval $[a, b]$, where x_i is the right endpoint of each equal subinterval, given $a = 1$ and $b = 3$.

[16pts]

Hint – Evaluate the limit:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(2x_i - 1 \right) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(2 \left(1 + \frac{i-1}{n} \right) - 1 \right) \frac{2-1}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(2 \frac{i-1}{n} + 1 \right) \frac{1}{n} = \lim_{n \rightarrow \infty} \left(\frac{2}{n^2} \sum_{i=1}^n (i-1) + \sum_{i=1}^n \frac{1}{n} \right) = \lim_{n \rightarrow \infty} \left(\frac{2}{n^2} \frac{(n-1)n}{2} + 1 \right) = \lim_{n \rightarrow \infty} (n-1) + 1 = 2$$

