PHY 3060 Homework assignments

Homework #1

Due in class Thursday August 27, 2009

From Schroeder's Thermal Physics do the following problems:

1.12, 1.15, 1.21, 1.23, & 1.27

Homework #2

Due in class Thursday Sept. 3, 2009

From Schroeder's Thermal Physics do the following problems:

1.31, 1.36, 1.38, 1.41, & 1.47

Homework #3

Due in class Thursday Sept. 10, 2009

From Schroeder's Thermal Physics do the following problems:

1.49, 1.60, 2.1, 2.6, 2.8

Homework #4

Due in class Thursday Sept. 17, 2009

From Schroeder's Thermal Physics do the following problems:

2.17, 2.19

Homework #5

Due in class Thursday Oct. 1, 2009

From Schroeder's Thermal Physics do the following problems:

2.33, 2.37, 3.1, 3.5

Homework #6

Due in class Thursday Oct. 8, 2009

From Schroeder's Thermal Physics do the following problems:

3.8, 3.10, 3.17, 3.28, 3.32, 3.36

Homework #7

From Schroeder's Thermal Physics do the following problems:

4.2, 4.3, 4.7, 4.8, 4.10

Homework #8 Practice problems

From Schroeder's Thermal Physics do the following problems:

4.24, 4.25, 4.26

Homework #9

From Schroeder's Thermal Physics do the following problems:

5.1, 5.2, 5.5, 5.32, 5.35, 5.42

Homework #10

Due in class Monday Nov. 16, 2009
From Schroeder's Thermal Physics do the following problems:
5.56, 5.60, 5.76, 5.89, 5.91

Homework #11

Due in class Thursday Nov. 19, 2009
From Schroeder's Thermal Physics do the following problems:
6.5, 6.10, 6.12

Homework #12

6.22b-f, 6.47, 7.9, 7.11, 7.13, 7.19

Problem 1.41. Th[illegible] To do so, put it in thermal contact with [illegible] know. As an example, suppose that a chunk of metal is immersed in boiling water (100°C), then is quickly transferred into a Styrofoam cup containing 250 g of water at 20°C. After a minute or so, the temperature of the contents of the cup is 24°C. Assume that during this time no significant energy is transferred between the contents of the cup and the surroundings. The heat capacity of the cup itself is negligible.

(a) How much heat is gained by the water?

(b) How much heat is lost by the metal?

(c) What is the heat capacity of this chunk of metal?

(d) If the mass of the chunk of metal is 100 g, what is its specific heat capacity?

Problem 1.47. Your 200 g cup of [illegible] is boiling hot. About how much ice should you add to bring it down to a comfortable sipping temperature of 65°C? (Assume that the ice is initially at −15°C. The specific heat capacity of ice is 0.5 cal/g·°C.)

Problem 1.49. Consider the combustion of one mole of H_2 with 1/2 mole of O_2 under standard conditions, as discussed in the text. How much of the heat energy produced comes from a decrease in the internal energy of the system, and how much comes from work done by the collapsing atmosphere? (Treat the volume of the liquid water as negligible.)

Problem 1.60. A frying pan is quickly heated on the stovetop to 200°C. It has an iron handle that is 20 cm long. Estimate how much time should pass before the end of the handle is too hot to grab with your bare hand. (Hint: The cross-sectional area of the handle doesn't matter. The density of iron is about 7.9 g/cm³ and its specific heat is 0.45 J/g·°C).

Problem 2.1. Suppose you flip four fair coins.

(a) Make a list of all the possible outcomes, as in Table 2.1.

(b) Make a list of all the different "macrostates" and their probabilities.

(c) [illegible] formula 2.6, and check that these results [illegible] force counting.

Problem 2.6. Calculate the multiplicity of an Einstein solid with 30 oscillators and 30 units of energy. (Do not attempt to list all the microstates.)

Problem 2.8. Consider a system of two Einstein solids, A and B, each containing 10 oscillators, sharing a total of 20 units of energy. Assume that the solids are weakly coupled, and that the total energy is fixed.

(a) How many different *macro*states are available to this system?

(b) How many different *micro*states are available to this system?

(c) Assuming that this system is in thermal equilibrium, what is the probability of finding all the energy in solid A?

Problem 1.12. Calculate the average volume per molecule for an ideal gas at room temperature and atmospheric pressure. Then take the cube root to get an estimate of the average distance between molecules. How does this distance compare to the size of a small molecule like N_2 or H_2O?

Problem 1.15. Estimate the average temperature of the air inside a hot-air balloon [illegible]. Assume that the total mass of the unfilled balloon and payload is 500 kg. What is the mass of the air inside the balloon?

Problem 1.21. D[illegible] area of the window is 0.5 m² and the hailstones hit it at [illegible] pressure do they exert on the window? [illegible] compare to the pressure of the atmosphere?

Problem 1.23. Calculate the total thermal energy in a liter of helium at room temperature and atmospheric pressure. Then repeat the calculation for a liter of air.

Problem 1.27. Give an example of a process in which [illegible] heat is added to a system, but its tem[illegible] opposite: a process in which heat is added to a system but its temperature does not change.

Problem 1.31. I[illegible] some helium in a cylinder with an initial volume of 1 liter [illegible] volume of 3 liters [illegible] pressure rises in direct proportion to its volume.

(a) Sketch a graph of pressure vs. volume for this process.

(b) Calculate the work done on the gas during this process, assuming that there are no "other" types of [illegible]

(c) Calculate the change in the helium's energy content during this process.

(d) Calculate the amount of heat added to or removed from the helium during this process.

(e) Describe what you might do to *cause* the pressure to rise as the helium expands.

Problem 1.36. In the course of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atm. (Air is mostly diatomic nitrogen and oxygen.)

(a) What is the final volume of this air after compression?

(b) How much work is done in compressing the air?

Problem 2.17. Use the methods of this section to derive a formula, similar to equation 2.21, for the multiplicity of an Einstein solid in the "low-temperature" limit, $q \ll N$.

Problem 2.19. Use Stirling's approximation to find an approximate formula for the multiplicity of a two-state paramagnet. Simplify this formula in the limit $N_\downarrow \ll N$ to obtain $\Omega \approx (Ne/N_\downarrow)^{N_\downarrow}$. This result should look very similar to your answer to Problem 2.17; explain why these two systems, in the limits considered, are essentially the same.

Problem 2.33. Use the Sackur-Tetrode equation to calculate the entropy of a mole of argon gas at room temperature and atmospheric pressure. Why is the entropy greater than that of a mole of helium under the same conditions?

Problem 2.37. Using the same method as in the text, calculate the entropy of mixing for a system of two monatomic ideal gases, A and B, whose relative proportion is arbitrary. Let N be the *total* number of molecules and let x be the fraction of these that are of species B. You should find

$$\Delta S_{\text{mixing}} = -Nk\left[x \ln x + (1-x)\ln(1-x)\right].$$

Check that this expression reduces to the one given in the text when $x = 1/2$.

Problem 3.1. Use Table 3.1 to compute the temperatures of solid A and solid B when $q_A = 1$. Then compute both temperatures when $q_A = 60$. Express your answers in terms of ϵ/k, and then in kelvins assuming that $\epsilon = 0.1$ eV.

q_A	Ω_A	S_A/k	q_B	Ω_B	S_B/k	Ω_{total}	S_{total}/k
0	1	0	100	2.8×10^{81}	187.5	2.8×10^{81}	187.5
1	300	5.7	99	9.3×10^{80}	186.4	2.8×10^{83}	192.1
2	45150	10.7	98	3.1×10^{80}	185.3	1.4×10^{85}	196.0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
11	5.3×10^{19}	45.4	89	1.1×10^{76}	175.1	5.9×10^{95}	220.5
12	1.4×10^{21}	48.7	88	3.4×10^{75}	173.9	4.7×10^{96}	222.6
13	3.3×10^{22}	51.9	87	1.0×10^{75}	172.7	3.5×10^{97}	224.6
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
59	2.2×10^{68}	157.4	41	3.1×10^{46}	107.0	6.8×10^{114}	264.4
60	1.3×10^{69}	159.1	40	5.3×10^{45}	105.5	6.9×10^{114}	264.4
61	7.7×10^{69}	160.9	39	8.8×10^{44}	103.5	6.8×10^{114}	264.4
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	1.7×10^{96}	221.6	0	1	0	1.7×10^{96}	221.6

Table 3.1. Macrostates, multiplicities, and entropies of a system of two Einstein solids, one with 300 oscillators and the other with 200, sharing a total of 100 units of energy.

Problem 3.5. Starting with the result of Problem 2.17, find a formula for the temperature of an Einstein solid [illegible] function of temperature to obtain $U = N\epsilon e^{-\epsilon/kT}$ (where ϵ is the size of an energy unit).

Problem 3.8. Starting with the result of Problem 3.5, [illegible] ity of an Einstein solid in the low-temperature limit. Sketch the predicted heat capacity as a function of temperature. (Note: Measurements of heat capacities of actual solids at low temperatures do not confirm the prediction that you will make in this problem. A more accurate model of solids at low temperatures is presented in Section 7.5.)

Problem 3.10. An ice cube (mass 30 g) at 0°C is left sitting on the kitchen table, where it gradually melts. The temperature in the kitchen is 25°C.

(a) Calculate the change in the [illegible]

(b) [illegible] entropy of the water (from the melted ice) as its temperature rises from 0°C to 25°C.

(c) Calculate the change in the entropy of the kitchen as it gives up heat to the melting ice/water.

(d) [illegible] universe during this process. Is the net change positi[illegible]

Problem 3.17. Verif[illegible] Table 3.2 (starting with $N_\uparrow = 98$).

$N_\uparrow$	$U/\mu B$	$M/N\mu$	Ω	S/k	$kT/\mu B$	C/Nk
100	−100	1.00	1	0	0	—
99	−98	.98	100	4.61	.47	.074
98	−96	.96	4950	8.51	.54	.310
97	−94	.94	1.6×10^5	11.99	.60	.365
⋮	⋮	⋮	⋮	⋮	⋮	⋮
52	−4	.04	9.3×10^{28}	66.70	25.2	.001
51	−2	.02	9.9×10^{28}	66.76	50.5	—
50	0	0	1.0×10^{29}	66.78	∞	—
49	2	−.02	9.9×10^{28}	66.76	−50.5	—
48	4	−.04	9.3×10^{28}	66.70	−25.2	.001
⋮	⋮	⋮	⋮	⋮	⋮	⋮
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible] elementary dipoles. Microscopic [illegible] pointing up, $N_\uparrow$. The multiplicity Ω is calculated from the combinatoric formula [illegible]

Problem 3.28. A liter of air, initially at room temperature and atmospheric pressure, is heated at constant pressure until it doubles [illegible] increase in its entropy during this process.

Problem 3.32. A cylinder contains one liter of air at room temperature (300 K) and atmospheric pressure (10^5 N/m^2). At one end of the cylinder is a massless piston, whose surface area is 0.01 m^2. Suppose that you push the piston in very suddenly, exerting a force of 2000 N. The piston moves only one millimeter, before it is stopped by an immovable barrier of some sort.

(a) How much work have you done on this system?

(b) How much heat has been added to the gas?

(c) Assuming that all the energy added goes into the gas (not the piston or cylinder walls), [illegible] increase?

(d) Use the thermodynamic identity to calculate the change in the entropy of [illegible]

Problem 3.36. Consider an Einstein solid for which [illegible]

(a) Show that the chemical potential is

$$\mu = -kT \ln\left(\frac{N+q}{N}\right).$$

(b) Discuss this result in the limits $N \gg q$ and $N \ll q$, concentrating on the question of how much S increases when another particle carrying no energy [illegible]

Problem 4.2. At a power plant that produces 1 GW (10^9 watts) of electricity, the steam turbines take in steam at a temperature of 500°C, and the waste heat is expelled into the environment at 20°C.

(a) What is the maximum possible efficiency of this plant?

(b) Suppose you develop a new material for making pipes and turbines, which allows the maximum steam temperature to be raised to 600°C. Roughly [illegible] if you sell the additional electricity for 5 cents per kilowatt hour? (Assume that the amount of fuel consumed at the plant is unchanged.)

Problem 4.3. A power plant produces 1 GW of electricity, at an efficiency of 40% (typical of today's coal-fired plants).

(a) At what rate does this plant expel waste heat into its environment?

(b) Assume first that the cold reservoir for this plant is a river whose flow rate [illegible]

(c) To avoid this "thermal pollution" of the river, the plant could instead be cooled by evaporation of river water. (This is more expensive, but in some areas it is environmentally preferable.) At what rate must the water evaporate? [illegible]

Problem 4.7. Why must you put an air conditioner in the window of a building

Problem 4.8. Can you cool off your kitchen by leaving the refrigerator door open? Explain.

Problem 4.10. Suppose that heat leaks into your kitchen refrigerator at an average rate of [illegible] watts. Assuming ideal operation, [illegible] draw from the wall?

Problem 4.24. Calculate the efficiency of a Rankine cycle that is modified from the parameters used in the text in each of the following three ways (one at a time), and comment briefly on the results: (a) reduce the maximum temperature to 500°C; (b) reduce the maximum pressure to 100 bars; (c) reduce the minimum temperature to 10°C.

Problem 4.25. In a real turbine, the entropy of the steam will increase somewhat. How will this affect the percentages of liquid and gas at point 4 in the cycle? How will the efficiency be affected?

Problem 4.26. A coal-fired power plant, with parameters similar to those used [illegible] of steam (in kilograms) that must pass through the turbine(s) each second.

Problem 5.1. Let the system [illegible] (kinetic only, neglecting atomic rest energies), entropy, [illegible] Express all answers in SI units.

Problem 5.2. Consider the production of ammonia from nitrogen and hydrogen,

$$N_2 + 3H_2 \longrightarrow 2NH_3,$$

at 298 K and 1 bar. From the values of ΔH and S [illegible] back [illegible] ΔG for this reaction [illegible] consistent with the value given in the table.

Problem 5.5. Consider a fuel cell that uses methane ("natural gas") as fuel. The reaction is

$$CH_4 + 2O_2 \longrightarrow 2H_2O + CO_2.$$

(a) Use the data at the back of this book to determine the values of ΔH and [illegible] Assume that the reaction [illegible] pressure.

(b) [illegible] the cell, for each mole of methane fuel?

(c) How much waste heat is produced, for each mole of methane fuel?

(d) The steps of this reaction are

[illegible]

at + electrode: $2O_2 + 8H^+ + 8e^-$ [illegible]

Problem 5.32. The density of ice is 917 kg/m^3.

(a) Use the Clausius-Clapeyron relation to explain why the slope of the phase boundary between water and ice is negative.

(b) How much pressure would you have to put on an ice cube to make it melt at $-1°$C?

(c) Approximately how deep under a glacier would you have to be before the weight of the ice above gives the pressure you found in part (b)? (Note that the pressure can be greater at some locations, as where the glacier flows over a protruding rock.)

(d) Make a rough estimate of the pressure under the blade of an ice skate, and calculate the melting temperature of ice at this pressure. Some authors [illegible] pressure under the blade melts the ice to create a thin layer of water. What do you think of this explanation?

Problem 5.35. The Clausius-Clapeyron relation 5.47 is a differential equation that can, in principle, be solved to find the shape of the entire phase-boundary curve. To solve it, however, you have to know how both L and ΔV depend on temperature and pressure. [illegible] can take L to be constant. Moreover, if one of the phases is a gas, you can usually neglect the volume of the [illegible] and just take ΔV to be the volume of the gas, expressed in terms of temperature and pressure using the ideal gas law. Making all these assumptions, [illegible] to obtain the following formula for the phase boundary [illegible]

$$P = (\text{constant}) \times e^{-L/RT}.$$

This result is called the **vapor pressure equation.** Caution: Be sure to use this formula only when all the assumptions just listed are valid.

Problem 5.42. [illegible] water vapor in the air is less than the equilibrium vapor pressure at the ambient temperature; this is [illegible] of water will spontaneously evaporate. The ratio of the partial pressure of water vapor to the equilibrium vapor pressure is called the relative humidity. When the relative humidity is 100%, so that water vapor in the atmosphere [illegible] diffusive equilibrium with a cup of liquid [illegible] the air is saturated. The dew point is the temperature at which the relative [illegible] for a given partial pressure of water vapor.

(a) Use the vapor pressure equation (Problem [illegible] to plot a graph of the vapor pressure of water from 0°C to 40°C. Notice that the vapor pressure approximately doubles for [illegible] temperature.

(b) The temperature on a certain [illegible] the relative humidity is 90%? [illegible]

Problem 5.56. [illegible] entropy of mixing of an ideal mixture has an infinite slope, when plotted vs. x, at $x = 0$ and $x = 1$.

Problem 5.60. Suppose

Problem 5.76. Seawater has a salinity of 3.5%, meaning that if you boil away a kilogram of seawater, when you're finished you'll have 35 g of solids (mostly NaCl) left in the pot. When dissolved, sodium chloride dissociates into separate Na^+ and Cl^- ions.

(a) Calculate the osmotic pressure difference between seawater and fresh water. Assume for simplicity that all the dissolved salts in seawater are NaCl.

(b) If you apply a pressure difference [illegible] get reverse osmosis: a flow of solvent out of the solution. This process [illegible] work required would be greater than the minimum [illegible]

Problem 5.89. The standard enthalpy change upon dissolving one mole of oxygen at 25°C is −11.7 kJ. Use this number and the van't Hoff equation (Problem 5.85) to calculate the equilibrium (Henry's law) constant for oxygen in water at 0°C and at 100°C. Discuss the results briefly.

Problem 5.91. When carbon dioxide "dissolves" in water, essentially all of it reacts to form carbonic acid, H_2CO_3:

$$CO_2(g) + H_2O(l) \longleftrightarrow H_2CO_3(aq)$$

The carbonic acid can then dissociate into H^+ [illegible]

$$H_2CO_3(aq) \longleftrightarrow H^+(aq) + HCO_3^-(aq).$$

(The table at the back of this book gives thermodynamic data for both of these reactions.) Consider a body of otherwise pure water (or perhaps a raindrop) that is in equilibrium with the atmosphere near sea level, where the partial pressure of carbon dioxide is 3.4×10^{-4} bar (or 340 parts per million). Calculate the [illegible] of carbonic acid and of bicarbonate ions in the water, and determine the pH of the solution. Note that even "natural" precipitation [illegible]

Problem 6.5. Imagine a particle that can be in only th[illegible] [illegible] This particle is in equilibrium with [illegible]

(a) [illegible] partition function for this particle.

(b) Calculate the probability for this [illegible] three states.

(c) Because the zero point for measuring energies is arbitrary, [illegible] well say that the energies of the three [illegible] using these numbers. Explain what changes and what doesn't.

Problem 6.10. A water molecule can vibrate [illegible] but the easiest type of vibration to excite is the "flexing" mode in which the hydrogen atoms move toward and away from each other but the HO bonds [illegible] of this mode are approximately harmonic, [illegible] for any quantum harmonic oscillator, the energy levels are $\frac{1}{2}hf$, $\frac{3}{2}hf$, $\frac{5}{2}hf$, and so on. None of these levels are degenerate.

(a) Calculate the probability [illegible] in its flexing ground state and in each of the first [illegible]

Problem 6.12. Cold interstellar molecular clouds often contain the molecule cyanogen (CN), whose first rotational excited states have an energy of 4.7×10^{-4} eV (above the ground state). There are actually three such excited states, all with the same energy. In 1941, studies of the absorption spectrum of starlight that passes through these molecular clouds showed that for every ten CN molecules that are in the ground state, approximately three others are in the three first excited states (that is, an average of one in each of these states). To account for this data, astronomers suggested that the molecules might be in thermal equilibrium with some "reservoir" with a well-defined temperature. What is that temperature?*

Problem 6.22. In most paramagnetic materials, the individual magnetic particles have more than two independent states (orientations). The number of independent states depends on the particle's angular momentum "quantum number" j, which must be a multiple of 1/2. For $j = 1/2$ there are just two independent states, as discussed in the text above and in Section 3.3. More generally, the allowed values of the z component of a particle's magnetic moment are

$$\mu_z = -j\delta_\mu,\ (-j+1)\delta_\mu,\ \ldots,\ (j-1)\delta_\mu,\ j\delta_\mu,$$

where δ_μ is a constant, equal to the difference in μ_z between one state and the next. (When the particle's angular momentum comes entirely from electron spins, δ_μ equals twice the Bohr magneton. When orbital angular momentum also contributes, δ_μ is somewhat different but comparable in magnitude. For an atomic nucleus, δ_μ is roughly a thousand times smaller.) Thus the number of states is $2j+1$. In the presence of a magnetic field B pointing in the z direction, the particle's magnetic energy (neglecting interactions between dipoles) is $-\mu_z B$.

(a) Prove the following identity for the sum of a finite geometric series:

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}.$$

(Hint: Either prove this formula by induction on n, or write the series as a [illegible]

(b) Show that the partition function of a single magnetic particle is

$$Z = \frac{\sinh[b(j+\frac{1}{2})]}{\sinh\frac{b}{2}},$$

where $b = \beta\delta_\mu B$.

(c) Show that the total magnetization of a system of N such particles is

$$M = N\delta_\mu\Big[\big(j+\tfrac{1}{2}\big)\coth\big[b\big(j+\tfrac{1}{2}\big)\big] - \tfrac{1}{2}\coth\tfrac{b}{2}\Big],$$

where coth is the hyperbolic cotangent, equal to cosh/sinh. Plot the quantity $M/N\delta_\mu$ vs. b, for a few different values of j.

(d) Show that the magnetization has the expected behavior as $T \to 0$.

(e) Show that the magnetization is proportional to $1/T$ (Curie's law) in the limit $T \to \infty$. (Hint: First show that $\coth x \approx \frac{1}{x} + \frac{x}{3}$ when $x \ll 1$.)

(f) Show that for $j = 1/2$, the result of part (c) reduces to the formula derived in the text for a two-state paramagnet.

Problem 7.9. Compute the quantum volume for an N_2 molecule at room temperature, and argue that a gas of such molecules at atmospheric pressure can be treated using Boltzmann statistics. At about what temperature would quantum statistics become relevant for this system (keeping the density constant and pretending that the gas does not liquefy)?

Problem 7.11. For a system of fermions at room temperature, compute the probability of a single-particle state being occupied if its energy is

- **(a)** 1 eV less than μ
- **(b)** 0.01 eV less than μ
- **(c)** equal to μ
- **(d)** 0.01 eV greater than μ
- **(e)** 1 eV greater than μ

Problem 7.13. For a system of bosons at room temperature, compute the average occupancy of a single-particle state and the probability of the state containing 0, 1, 2, or 3 bosons, if the energy of the state is

- **(a)** 0.001 eV greater than μ
- **(b)** 0.01 eV greater than μ
- **(c)** 0.1 eV greater than μ
- **(d)** 1 eV greater than μ

Problem 7.19. Each atom in a chunk of copper contributes one conduction electron. Look up the density and atomic mass of copper, and calculate the Fermi energy, the Fermi temperature, the degeneracy pressure, and the contribution of the degeneracy pressure to the bulk modulus. Is room temperature sufficiently low to treat this system as a degenerate electron gas?