

## PHY 3060 Homework assignments

### Homework #1

Due in class Thursday August 27, 2009

From Schroeder's Thermal Physics do the following problems:

1.12, 1.15, 1.21, 1.23, & 1.27

### Homework #2

Due in class Thursday Sept. 3, 2009

From Schroeder's Thermal Physics do the following problems:

1.31, 1.36, 1.38, 1.41, & 1.47

### Homework #3

Due in class Thursday Sept. 10, 2009

From Schroeder's Thermal Physics do the following problems:

1.49, 1.60, 2.1, 2.6, 2.8

### Homework #4

Due in class Thursday Sept. 17, 2009

From Schroeder's Thermal Physics do the following problems:

2.17, 2.19

### Homework #5

Due in class Thursday Oct. 1, 2009

From Schroeder's Thermal Physics do the following problems:

2.33, 2.37, 3.1, 3.5

### Homework #6

Due in class Thursday Oct 8 2009

3.8, 3.10, 3.17, 3.28, 3.32, 3.36

### Homework #7

From Schroeder's Thermal Physics do the following problems:

4.2, 4.3, 4.7, 4.8, 4.10

### Homework #8 Practice problems

4.24, 4.25, 4.26

### Homework #9

From Schroeder's Thermal Physics do the following problems:

5.1, 5.2, 5.5, 5.32, 5.35, 5.42

Due in class Monday Nov. 16, 2009

From Schroeder's Thermal Physics do the following problems:  
5.56, 5.60, 5.76, 5.89, 5.91

*Homework #11*

Due in class Thursday Nov. 19, 2009

From Schroeder's Thermal Physics do the following problems:  
6.5, 6.10, 6.12

*Homework #12*

6.22b-f, 6.47, 7.9, 7.11, 7.13, 7.19

**Problem 1.41.** You are given a chunk of metal that has been heated in boiling water (100°C), then is quickly transferred into a Styrofoam cup containing 250 g of water at 20°C. After a minute or so, the temperature of the contents of the cup is 24°C. Assume that during this time no significant energy is transferred between the contents of the cup and the surroundings. The heat capacity of the cup itself is negligible.

- How much heat is gained by the water?
- How much heat is lost by the metal?
- What is the heat capacity of this chunk of metal?
- If the mass of the chunk of metal is 100 g, what is its specific heat capacity?

**Problem 1.47.** Your 200 g cup of tea is boiling hot. About how much ice should you add to bring it down to a comfortable sipping temperature of 15°C? (The specific heat capacity of ice is 0.5 cal/g·°C.)

**Problem 1.49.** Consider the combustion of one mole of  $H_2$  at 1 atm and 25°C under standard conditions, as discussed in the text. How much of the heat energy produced comes from a decrease in the internal energy of the system, and how much comes from work done by the collapsing atmosphere? (The heat capacity of  $H_2$  is 5/2 R.)

**Problem 1.60.** A frying pan is quickly heated on the stove to 200°C. The iron handle that is 20 cm long. Estimate how long it takes for the end of the handle to be too hot to grab with your bare hand. (Hint: The cross-sectional area of the handle doesn't matter. The density of iron is about 7.8 g/cm<sup>3</sup> and its specific heat is 0.45 J/g·°C.)

**Problem 2.1.** Suppose you flip four fair coins.

- Make a list of all the possible outcomes.
- Make a list of all the different "microstates" that correspond to each of the possible outcomes.

and check that these results

agree with your force counting.

**Problem 2.6.** Calculate the multiplicity of an Einstein solid with 20 oscillators and 40 units of energy. (You should be able to list all the microstates.)

**Problem 2.8.** Consider a system of two Einstein solids, A and B, each containing 10 oscillators sharing a total of 20 units of energy. Assume that the solids are weakly coupled, and that the total energy is fixed.

- How many different microstates are available to this system?
- How many different microstates are available to this system?
- Assuming that this system is in thermal equilibrium, what is the probability of finding all the energy in solid A?

**Problem 1.13.** Calculate the average volume per molecule for an ideal gas at room temperature and atmospheric pressure. Then take the cube root to get an estimate of the average distance between molecules. How does this distance compare to the size of a small molecule like  $N_2$  or  $H_2O$ ?

**Problem 1.15.** Estimate the average temperature of the air inside a hot air balloon. Assume that the total mass of the balloon and payload is 500 kg. What is the mass of the air inside the balloon?

**Problem 1.21.** Drizzle is falling from a cloud at a rate of 1 mm per hour. The rain is 0.5 mm in diameter. The rain falls at a speed of 20 m/s. What is the pressure exerted on the ground by the rain? How does this compare to the pressure in the atmosphere?

**Problem 1.23.** Calculate the total thermal energy in a liter of air at room temperature and atmospheric pressure. Then repeat the calculation for a liter of air.

**Problem 1.27.** Give an example of a process in which heat is added to a system but its temperature does not change.

**Problem 1.31.** Some helium in a cylinder with an initial volume of 1 liter

- Sketch a graph of pressure vs. volume for this process.
- Calculate the work done on the gas during this process, assuming that there are no "other" types of molecules.
- Calculate the change in internal energy of the helium during this process.
- Estimate the amount of heat added to or removed from the helium during this process.
- Describe what you might do to cause the pressure to rise as the helium expands.

**Problem 1.36.** In the course of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atm. (Air is made up of nitrogen and oxygen.)

- What is the final volume of this air after compression?

**Problem 2.17.** Use the methods of this section to derive a formula, similar to equation 2.21, for the multiplicity of an Einstein solid in the "low temperature" limit,  $a \ll N$ .

**Problem 2.19.** Use the same method as in Problem 2.17 to derive a formula for the multiplicity of a two-oscillator Einstein solid in the "low temperature" limit,  $a \ll N$ .

answer to Problem 2.17; explain why these two formulas are essentially the same.

**Problem 2.33.** Use the Sackur-Tetrode equation to calculate the entropy of a mole of argon gas at room temperature and atmospheric pressure. Why is the entropy greater than that of a mole of helium under the same conditions?

**Problem 2.37.** Using the same method as in the text, calculate the entropy of mixing for a system of two monatomic ideal gases,  $A$  and  $B$ , whose relative proportion is arbitrary. Let  $N$  be the total number of molecules and let  $x$  be the fraction of those that are of species  $A$ .

Check that this expression reduces to the entropy of a single gas when  $x = 1$ . Then compute both temperature and answers in terms of  $e/k$ .

$q_A$	$\Omega_A$	$S_A/k$	$q_B$	$\Omega_B$	$S_B/k$	$\Omega_{\text{total}}$	$S_{\text{total}}/k$
1	300	5.7	99	$9.3 \times 10^{80}$	186.4	$2.8 \times 10^{83}$	192.1
2	45150	10.7	98	$3.1 \times 10^{80}$	185.3	$1.4 \times 10^{85}$	196.0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
11	$5.3 \times 10^{19}$	45.4	89	$1.1 \times 10^{76}$	175.1	$5.0 \times 10^{95}$	220.5
12	$1.1 \times 10^{20}$	46.7	88	$3.4 \times 10^{76}$	173.9	$4.7 \times 10^{96}$	222.6
13	$3.3 \times 10^{22}$	51.9	87	$1.0 \times 10^{75}$	172.7	$3.5 \times 10^{97}$	224.6
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
50	$2.2 \times 10^{68}$	155.1	51	$2.0 \times 10^{46}$	117.0	$6 \times 10^{114}$	264.4
60	$1.3 \times 10^{92}$	159.1	40	$5.3 \times 10^{45}$	105.5	$3.6 \times 10^{114}$	264.4
61	$7.7 \times 10^{69}$	160.9	39	$8.8 \times 10^{44}$	103.5	$6.8 \times 10^{114}$	264.4
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	$1.7 \times 10^{96}$	221.6	0	1	0	$1.7 \times 10^{96}$	221.6

Table 3.1. Macrostates, multiplicities, and entropies of a system of two Einstein solids, each with 100 units of energy, sharing a total of 100 units of energy.

**Problem 3.5.** Starting with the result of Problem 2.17, find a formula for the temperature of an Einstein solid as a function of temperature to obtain  $\langle E \rangle = N \epsilon e^{-\epsilon/R T}$  (where  $\epsilon$  is the size of an energy unit).

**Problem 3.6.** Starting with the result of Problem 2.5, sketch the heat capacity of an Einstein solid in the low-temperature limit. Sketch the predicted heat capacity as a function of temperature. (Note: Measurements of heat capacities of actual solids at low temperatures do not confirm the prediction that you will make in this problem. A more accurate model of solids at low temperatures is presented in Section 7.5.)

**Problem 3.10.** An ice cube (mass 30 g) at  $0^\circ\text{C}$  is left sitting on the kitchen table, where it gradually melts. The temperature in the kitchen is  $25^\circ\text{C}$ .

- (a) Calculate the change in the entropy of the water (from the melted ice) as its temperature rises from  $0^\circ\text{C}$  to  $25^\circ\text{C}$ .
- (c) Calculate the change in the entropy of the kitchen as it gives up heat to the melting ice/water.

Is the net change positive or negative during this process?

$N_{\uparrow} = 98$ .

$N_{\uparrow}$	$\langle E \rangle / \mu R$	$M / N_{\uparrow} \mu$	$Q$	$C / \mu$	$\langle E \rangle / \mu R$	$C / \mu R$
100	-100	1.00	1	0	0	—
99	-98	.98	100	4.61	.47	.074
98	-96	.96	4950	8.51	.54	.310
97	-94	.94	$1.6 \times 10^5$	11.99	.60	.365
⋮	⋮	⋮	⋮	⋮	⋮	⋮
52	-4	.04	$9.3 \times 10^{28}$	66.70	25.2	.001
51	-2	.02	$9.9 \times 10^{28}$	66.76	50.5	—
50	0	0	$1.0 \times 10^{29}$	66.78	$\infty$	—
49	2	-.02	$9.9 \times 10^{28}$	66.76	-50.5	—
48	4	-.04	$9.3 \times 10^{28}$	66.70	-25.2	.001
⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	100	1.00	1	0	0	—

licity  $\Omega$  is calculated from the combinatoric formula  $\Omega = \frac{N!}{N_{\uparrow}! (N - N_{\uparrow})!}$ .

**Problem 3.28.** A liter of air, initially at room temperature and atmospheric pressure, is heated at constant pressure until it undergoes an increase in its entropy during this process.

**Problem 3.32.** A cylinder contains one liter of air at room temperature (300 K) and atmospheric pressure ( $10^5 \text{ N/m}^2$ ). At one end of the cylinder is a massless piston, whose surface area is  $0.01 \text{ m}^2$ . Suddenly, exerting a force of 2000 N. The piston moves only one millimeter, before it is stopped by an immovable barrier of some sort.

- (a) How much work have you done on this system?
- (b) How much heat has been added to the gas?

Use the thermodynamic identity to calculate the change in the entropy of the gas.

**Problem 3.33.** Consider an Einstein solid...

(a) Show that the chemical potential is

$$\mu = -kT \ln\left(\frac{N+q}{N}\right).$$

(b) Discuss the behavior of this chemical potential as  $N \rightarrow \infty$  and  $N \rightarrow 0$ , concentrating on the question of how much  $q$  increases when another particle carrying no energy...

**Problem 4.1** At a power plant that produces 1 GW (10<sup>9</sup> watts) of electricity the steam turbines take in steam at a temperature of 500°C, and the waste heat is expelled into the environment at 20°C.

- (a) What is the maximum possible efficiency of this plant?
- (b) Suppose you have a new supermaterial for making a turbine which allows the maximum steam temperature to be raised to 600°C. Roughly how much additional electricity for 5 cents per kilowatt hour? (Assume that the amount of fuel consumed at the plant is unchanged.)

**Problem 4.2** A power plant produces 1 GW of electricity at an efficiency of 40% (typical of today's coal-fired plants).

- (a) At what rate does this plant expel waste heat into its environment?
- (b) Assume first that the cold reservoir for this plant is a river whose flow rate is 100 m<sup>3</sup>/s. At what rate must the water be heated?
- (c) To avoid this "thermal pollution" of the river, the plant could instead be cooled by evaporation of river water. (This is more expensive, but in some areas it is environmentally preferable.) At what rate must the water be evaporated?

**Problem 4.8.** Can you cool off your kitchen by leaving the refrigerator door open? Explain.

**Problem 4.10.** Suppose that heat leaks into your kitchen refrigerator at an average rate of 500 watts. Assuming normal operation, how much power does it draw from the wall?

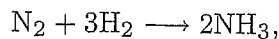
**Problem 4.24.** Calculate the efficiency of a Rankine cycle that is modified from the parameters used in the text in each of the following three ways (one at a time): (a) condensation is at the critical temperature (c) reduce the maximum temperature to 500°C; (b) reduce the maximum pressure to 180 bars; (d) reduce the minimum temperature to 10°C.

**Problem 4.25.** In a real turbine, the entropy of the steam will increase somewhat. How will this affect the percentages of liquid and gas at point 4 in the cycle? How will the efficiency be affected?

**Problem 4.26.** A coal-fired power plant with parameters similar to those used in the text is operating at 200°C. The condenser is at 30°C. The turbine inlet temperature is 500°C. (a) How much heat must pass through the turbine (a) each second?

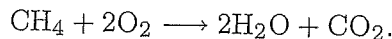
**Problem 5.1.** Consider the reaction  $\text{N}_2 + 3\text{H}_2 \rightarrow 2\text{NH}_3$ . Express all answers in SI units.

**Problem 5.2.** Consider the production of ammonia from nitrogen and hydrogen,



at 200 K and 1 bar. The standard enthalpy of formation of  $\text{NH}_3$  is given in the table given in the table.

**Problem 5.5.** Consider a fuel cell that uses methane ("natural gas") as fuel. The reaction is



(a) Use the data at the back of this book to determine the values of  $\Delta H^\circ$  and  $\Delta G^\circ$  for the reaction.

(b) How much electrical work can be done by the cell for each mole of methane fuel?

(c) How much waste heat is produced for each mole of methane fuel?

(d) The steps of this reaction are



**Problem 5.32.** The density of ice is  $917 \text{ kg/m}^3$ .

- (a) Use the Clausius-Clapeyron relation to explain why the slope of the phase boundary between water and ice is negative.
- (b) How much pressure would you have to put on an ice cube to make it melt at  $-1^\circ\text{C}$ ?
- (c) Approximately how deep under a glacier would you have to be before the weight of the ice above gives the pressure you found in part (b)? (Note that the pressure can be greater at some locations, as where the glacier flows over a protruding rock.)

- (a) make a rough estimate of the pressure under the blade of an ice skate, and calculate the melting temperature of ice at this pressure. Suppose the pressure under the blade melts the ice to create a thin layer of water. What do you think of this explanation?

**Problem 5.35.** The Clausius-Clapeyron relation 5.47 is a differential equation that can, in principle, be solved to find the shape of the entire phase-boundary curve. To solve it, however, you have to know how to handle the volume and temperature and pressure terms.

Making all these assumptions, use the ideal gas law.

$$P = (\text{constant}) \times e^{-L/RT}.$$

This result is called the **vapor pressure equation**. Caution: Be sure to use this formula only when all the assumptions just listed are valid.

**Problem 5.42.**

water vapor pressure at the ambient temperature. This is the rate at which water evaporates. The ratio of the partial pressure of water vapor to the equilibrium vapor pressure is called the **relative humidity**. When the relative humidity is 100%, so that water vapor in the atmosphere is in equilibrium with a cup of liquid water.

The **dew point** is the temperature at which the relative humidity is 100%.

- (a) Use the vapor pressure equation to plot a graph of the relative humidity of water from  $0^\circ\text{C}$  to  $40^\circ\text{C}$ . Notice that the vapor pressure approximately doubles for every  $10^\circ\text{C}$  increase in temperature.
- (b) The temperature on a certain day is  $20^\circ\text{C}$  and the relative humidity is 60%. What is the dew point?

**Problem 5.50.** For a binary mixture, the partial pressure of an ideal mixture has an infinite slope at  $x = 0$  and  $x = 1$ .

**Problem 5.51.** Suppose

**Problem 5.78.** Seawater has a salinity of 3.5%, meaning that if you boil away a kilogram of seawater, when you're finished you'll have 35 g of solids (mostly NaCl) left in the pot. When dissolved, sodium chloride dissociates into separate  $\text{Na}^+$  and  $\text{Cl}^-$  ions.

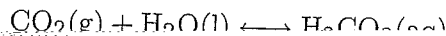
(a) Calculate the osmotic pressure difference between seawater and fresh water. Assume for simplicity that all the dissolved salts in seawater are NaCl.

(b) If you apply a pressure difference across a thin wall separating the two

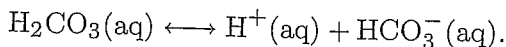
not reverse osmosis, a flow of solvent out of the solution. This is the

**Problem 5.89.** The standard enthalpy change upon dissolving one mole of oxygen at  $25^\circ\text{C}$  is  $-11.7\text{ kJ}$ . Use this number and the van't Hoff equation (Problem 5.85) to calculate the equilibrium (Henry's law) constant for oxygen in water at  $100^\circ\text{C}$ . Discuss the results briefly.

**Problem 5.91.** When carbon dioxide "dissolves" in water, essentially all of it reacts to form carbonic acid,  $\text{H}_2\text{CO}_3$ .



The carbonic acid can then



(The table at the back of this book gives thermodynamic data for both of these reactions.) Consider a body of otherwise pure water (or perhaps a raindrop) that

is in contact with the atmosphere near sea level, where the partial pressure of carbon dioxide is  $3.4 \times 10^{-4}\text{ bar}$  (or 340 parts per million). Calculate the concentration of carbonic acid and of bicarbonate ions in the water, and determine the pH of the solution. Note that even "natural" precipitation

**Problem 6.5.** Imagine a particle that can be in only the

lowest energy state. The particle is in equilibrium with

the environment. Calculate the probability for this particle.

(b) Calculate the probability for this

(c) Because the zero point for measuring energies is arbitrary,

we say that the energies of the three

**Problem 6.11.** A water molecule can vibrate in

type of vibration to excite is the "bending" mode, in which the oxygen

toward and away from each other but the H-O bonds

of this mode are approximately harmonic.

for any quantum harmonic oscillator, the energy levels are  $\frac{1}{2}hf$ ,  $\frac{3}{2}hf$ ,  $\frac{5}{2}hf$ , and so

on. None of these levels are degenerate.

(a) Calculate the probability of

state and in each of the first

**Problem 6.12.** Cold-interstellar molecular clouds often contain the molecule cyanogen (CN), whose first rotational excited states have an energy of  $4.7 \times 10^{-4}$  eV (above the ground state). There are actually three such excited states, all with the same energy. In 1941, studies of the absorption spectrum of starlight that passes through these molecular clouds showed that for every ten CN molecules that are in the ground state, approximately three others are in the three first excited states (that is, an average of one in each of these states). To account for this data, one would expect that the molecules might be in thermal equilibrium with the radiation field. What is that temperature?\*

more more than the independent states (directions), this number of independent states depends on the particle's angular momentum "quantum number"  $j$ , which must be a multiple of  $1/2$ . For  $j = 1/2$  there are just two independent states, as discussed in the text above and in Section 3.3. More generally, the allowed values of the  $z$  component of a particle's magnetic moment are

$$\mu_z = -j\delta\mu, (-j+1)\delta\mu, \dots, (j-1)\delta\mu, j\delta\mu, \dots$$

where  $\delta\mu$  is a constant, equal to the difference in  $\mu_z$  between one state and the next. (When the particle's angular momentum comes entirely from electron spin,  $\delta\mu$  equals twice the Bohr magneton. When orbital angular momentum also contributes,  $\delta\mu$  is somewhat different but comparable in magnitude. For an atomic nucleus,  $\delta\mu$  is roughly a thousand times smaller.) Thus the number of states is  $2j+1$ . In the presence of a magnetic field  $B$  pointing in the  $z$  direction, the particle's magnetic energy (neglecting interactions between dipoles) is  $-\mu_z B$ .

(a) Prove the following identity for the sum of a finite geometric series:

$$1 + x + x^2 + \dots + x^n = \frac{1 - x^{n+1}}{1 - x}.$$

(Hint: Either prove this formula by induction on  $n$ , or write the series as a

$$Z = \frac{\sinh[b(j + \frac{1}{2})]}{\sinh \frac{b}{2}},$$

where  $b = \beta\delta\mu B$ .

(c) Show that the total magnetization of a system of  $N$  such particles is

$$M = N\delta\mu \left[ \frac{1}{2} - \frac{1}{2} \coth \left( \frac{b}{2} \right) \right]$$

quantity  $M/N\delta\mu$  vs  $b$  for a few different values of  $j$ .

(d) Show that the magnetization has the Curie-law behavior  $\rightarrow 0$  as  $T \rightarrow \infty$ .

Show that the magnetization is proportional to  $1/T$  (Curie's law) in the limit  $T \rightarrow \infty$ . (Hint: First show that  $\coth x \approx \frac{1}{x} + \frac{x}{3}$  when  $x \ll 1$ .)

**Problem 7.9.** Compute the quantum volume for an  $\text{N}_2$  molecule at room temperature and argue that a gas of such molecules at atmospheric pressure can be treated as a classical gas. At what temperature would quantum statistics become relevant (assume that the gas does not liquify)?

**Problem 7.10.** For a system of fermions at room temperature, compute the probability of a single-particle state being occupied if its energy is

- (a) 1 eV less than  $\mu$
- (b) 0.01 eV less than  $\mu$
- (c) equal to  $\mu$
- (d) 0.01 eV greater than  $\mu$
- (e) 1 eV greater than  $\mu$

**Problem 7.13.** For a system of bosons at room temperature, compute the average occupancy of a single-particle state and the probability of the state containing 0,

1, 2, or 3 bosons, if the energy of the state is

- (a) 0.001 eV greater than  $\mu$
- (b) 0.01 eV greater than  $\mu$
- (c) 0.1 eV greater than  $\mu$
- (d) 1 eV greater than  $\mu$

**Problem 7.15.** Each atom in a chunk of copper contributes one conduction electron. Look up the density and atomic mass of copper and calculate the Fermi

energy. Treat the conduction electrons as a degenerate electron gas and the contribution of

the conduction electrons to the heat capacity of the bulk material. Is room temperature sufficiently low

to treat this system as a degenerate electron gas?